THE MOVING-COIL GALVANOMETER AND THE APPLICATION

OF A DAMPED COIL TO THE MEASUREMENT

OF MAGNETIC FIELDS

A THESIS

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

of the

UNIVERSITY OF MINNESOTA

by

PAUL E. KLOPSTEG

in partial fulfillment of the requirements for

the degree of

MASTER OF ARTS

June 12, 1913.

MOM
81569

THE MOVING-COIL GALVANOMETER AND THE APPLICATION OF A DAMPED COIL TO THE MEASUREMENT OF MAGNETIC FIELDS.

INTRODUCTION

During the past twenty years much work has been done both on the theory and on the practical application of the moving-coil galvanometer. The work was the natural result of the fact that the moving-coil galvanometer, on account of its almost absolute independence of external or stray magnetic fields, is fast replacing the moving magnet type.

One of the earliest papers relating in part to the D'Arsonval galvanometer appeared in 1890, its authors being Ayrton, Mather and Sumpner[1]; another paper by Ayrton and Mather[2] was published in 1898. Both the

[1] Phil. Mag. 30 (5), 1890, p. 73.
[2] Phil. Mag. 46 (5), 1898, p. 351.

papers referred to deal chiefly with the sensitivities of the various types of moving-coil galvanometers which then existed; the second paper also takes up some of the equations of motion of the coil. Diesselhorst[1] traces out a few of the results drawn from the solution of the general differential equation, and mentions the desirability of the condition of critical damping of the coil. O. M. Stewart[2] takes up the just aperiodic case in detail, and shows that whatever may be the coefficient of damping in a ballistic galvanometer, the throw remains proportional to the quantity of electricity discharged through the coil. The critically damped moving-coil galvanometer is discussed in detail by Jaeger[3]; sensitivities both for deflection and ballistic work are mentioned and formulas for calculation of constants given. White[4] and Diesselhorst[5] give equations

---

[1]Ann. d. Phys. 9, 1902, p. 458.
[2]Phys. Rev. 16, 1903, p. 158.
[3]Zschr. f. Instrumentenkunde, 23, 1903, pp. 261 and 353; Ann. d. Phys. 21, 1906, p. 64.
[4]Phys. Rev. 19, 1904, p. 305; Phys. Rev. 23, 1906, p. 382.
[5]Zschr. f. Instrumentenkunde, 31, 1911, p. 247.

for the design of galvanometers. A comprehensive work on the practical applications of the instrument appears in two papers by A. Zeleny[1]. In the first paper directions are given as to precautions which should be observed in using the ballistic moving-coil galvanometer as an instrument of precision. The second paper deals with several of the eccentricities of the instrument, "zero shift" and "deflection hysteresis."

Notwithstanding the progress which has been made toward a fuller understanding of the moving-coil galvanometer by virtue of the work which has been directed toward a solution of the problems which arise in connection with it, some problems still remain to be solved.

A careful search in many periodicals and texts indicates that up to the present time there has never appeared under a single heading a comprehensive discussion of the various types of motion which a galvanometer coil may have, nor a complete review of the theory of

---

[1]Phys. Rev. 23, 1906, p. 399; 32, 1911, p. 297.

the motions. These types—simple harmonic, damped periodic and aperiodic—have been repeatedly treated both in the theory of and the application to other physical problems, and individually in the case of galvanometers.

It is aimed in this paper to give a complete review of the various types of motion which the galvanometer coil may have, basing the discussion in each case upon the equation of the motion and its interpretation when applied to the galvanometer. This will lead up to a method of measuring magnetic fields by observations upon the motion of a damped coil in a magnetic field. Finally, a few of the peculiarities in the behavior of the moving coil and their causes will be mentioned.

A) REVIEW OF THE GALVANOMETER THEORY.

Case I. Simple Harmonic Motion.

Simple harmonic, or undamped periodic motion is never realized in a galvanometer, since it requires perfect elasticity in the suspension fibers, and no resistance of any kind to oppose the motion of the coil. The conditions may be approximated if the coil is suspended in a vacuum in zero magnetic field.

The case is of interest because from it comparisons may be drawn with the other types of motion.

The differential equation of the motion is

$$I_0 \frac{d^2\theta}{dt^2} = - q^2\theta, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where I_0 is the moment of inertia of the coil, and

q^2 is the torsional moment per unit angle of the fiber.

The motion which is represented by the equation is such that the acceleration is always toward the center, or position of rest, and that at any instant, t, the acceleration toward the position of rest is proportional to the angle of displacement from this position. The equation is solved by multiplying both sides of the equation by $2\frac{d\theta}{dt}$ and integrating. The solution is

$$t = \sqrt{\frac{I_0}{q^2}}\,(\arc \sin \frac{q\theta}{C_1}) + C_2,$$

or, in terms of t and the constants,

$$\theta = \frac{C_1}{q}\left[\sin\sqrt{\frac{q^2}{I_0}}\,(t - C_2)\right].$$

To determine the values of the constants of integration the following initial conditions are assumed: that the coil is initially at rest, and that at a given instant $t = 0$ a quantity of electricity Q is discharged through the coil. It is thereby given an angular velocity which is proportional to the quantity of electricity discharged, say kQ radians per second, where k has the value $(nlrH)/I_0$. In this expression H is the strength of the magnetic field, l the length of the coil, r its width and n the number of its turns. It is further assumed that the whole quantity Q is discharged before the coil has moved appreciably. With these assumptions the initial conditions are

$$t = 0, \quad \theta = 0, \quad d\theta/dt = kQ.$$

With these values the equation for θ becomes

$$\theta = kQ\sqrt{\frac{I_0}{q^2}}\sin t\sqrt{\frac{q^2}{I_0}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The graph of this equation is shown in Fig. 1. It is the ordinary sine curve; each amplitude is exactly the same in magnitude as the one preceding, and the motion continues indefinitely. The constants which were used in plotting this curve are: $I_0 = 3.84$; $q^2 = 1.60$; $n = 384$; $l = 5$; $r = 2$; $H = 400$; $Q = 8 \times 10^{-7}$.

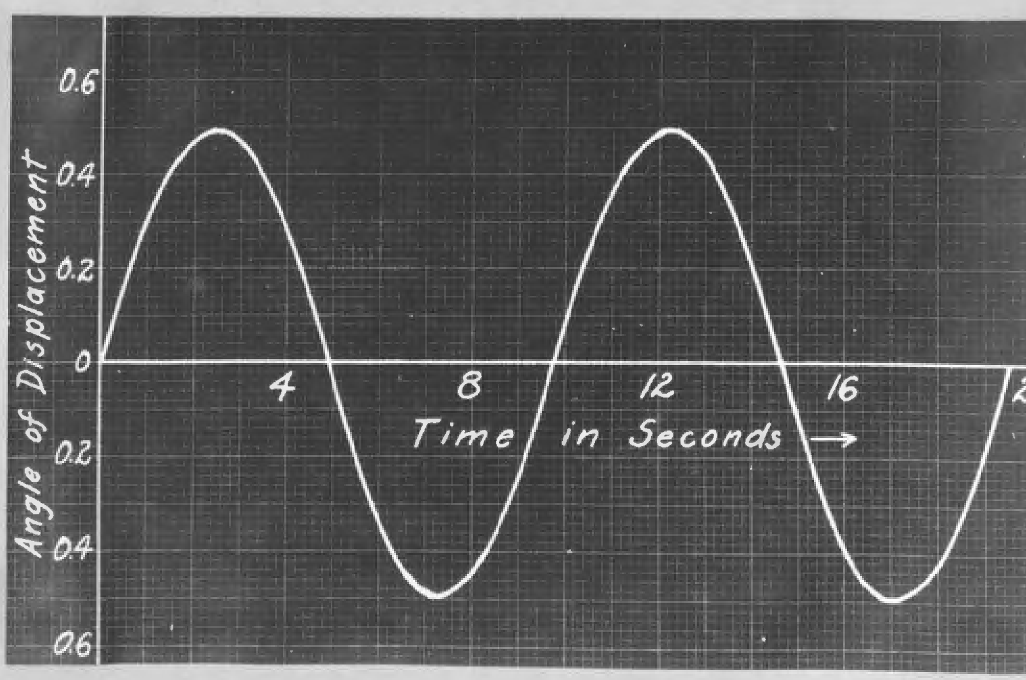

Fig. 1.

If the damping coefficient is not zero, i.e., if the coil in its motion encounters resistance, then some of the energy which it possesses by virtue of the impulse which set it swinging will be transformed into heat energy in overcoming the resistance. In Case I, where no damping was present, no energy goes into heat: there is only a periodic transformation from kinetic to potential energy, and vice versa. With damping present, that part of the energy which has been transformed into heat can no longer affect the motion of the coil; the sum of potential and kinetic energy is continuously diminishing; each amplitude is smaller than the one preceding. Or, the resistance to motion may be so great that the motion loses its periodic characteristic entirely. These cases have their analogies in the motion of a pendulum swinging, say, in air, in which case the motion remains periodic; then in some viscous medium like syrup, in which the motion becomes aperiodic. Between the moderately damped periodic and the very strongly damped aperiodic motions there is a "transitional" case. When this condition obtains, the motion is said to be <u>just</u> aperiodic, or critically damped. The different types of motion mentioned will now be taken up in greater detail in connection with the equations which represent them.

The differential equation for any kind of damped motion which a galvanometer coil may have when suspended in the usual way, is[1]

$$I_0\frac{d^2\theta}{dt^2} + 2f\frac{d\theta}{dt} + q^2\theta = 0. \; . \; . \; . \; . \; . \; . \; . \; (3)$$

In this equation I_0, q^2, θ and t have the same meanings as in Case I, and 2f is the coefficient of damping[2]. The assumption is made that the damping due to air friction and viscosity of the suspensions varies

[1]This equation with other applications or from the mathematical standpoint alone has been much used as an illustration of a linear differential equation of the second order. See, for example, Mellor, Higher Mathematics for Students of Chemistry and Physics, 404.

[2]As distinguished from the "damping factor" which is used to correct the throws from a ballistic galvanometer for damping.

directly as the angular velocity of the coil; that this is actually the rate of variation in the portion of damping which is due to eddy currents and induced currents will be shown in another section of this paper.

The equation is solved by substituting in it the integrating factor $\theta = \varepsilon^{mt}$; from this substitution results the auxiliary equation

$$I_0 m^2 + 2fm + q^2 = 0,$$

the roots of which are

$$-\frac{f}{I_0} + \sqrt{\frac{f^2}{I_0^2} - \frac{q^2}{I_0}} \quad \text{and} \quad -\frac{f}{I_0} - \sqrt{\frac{f^2}{I_0^2} - \frac{q^2}{I_0}}.$$

Let $\frac{f}{I_0} = a$, and $\sqrt{\frac{f^2}{I_0^2} - \frac{q^2}{I_0}} = \beta$. The general solution then is

$$\theta = \varepsilon^{-at}(C_1 \varepsilon^{\beta t} + C_2 \varepsilon^{-\beta t}) \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

The three cases which have before been mentioned arise out of the general solution:

Case II. Damped periodic motion; β is imaginary, $I_0 q^2 > f^2$.

Case III. Critically damped motion; $\beta = 0$, $I_0 q^2 = f^2$.

Case IV. Overdamped motion; β is real, $I_0 q^2 < f^2$.

Case II. Damped Periodic Motion.

If β is imaginary, equation (4) becomes

$$\theta = \varepsilon^{-at}(A \cos bt + B \sin bt), \quad . \ . \ . \ . \ (5)$$

in which $b = \sqrt{\frac{q^2}{I_0} - \frac{f^2}{I_0^2}}$.

To evaluate the constants, the same initial conditions may be imposed as under Case I; i.e.,

$$t = 0, \ \theta = 0, \ d\theta/dt = 0.$$

With these substitutions, eq. (5) becomes[1]

$$\theta = \varepsilon^{-at}(\frac{kQ}{b} \sin bt). \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (6)$$

This equation represents the angular position θ of the coil in a moderately damped galvanometer at any instant t after the quantity of electricity Q has been discharged through it. The curve of the equation is given in Fig. 2. In plotting the curve, the same constants were assumed as for the curve of Fig. 1, in addition to which the coefficient of damping was taken as 1.09.

---

[1] Putting f = 0 in this equation, it becomes identical with eq. (2), that of simple harmonic motion.

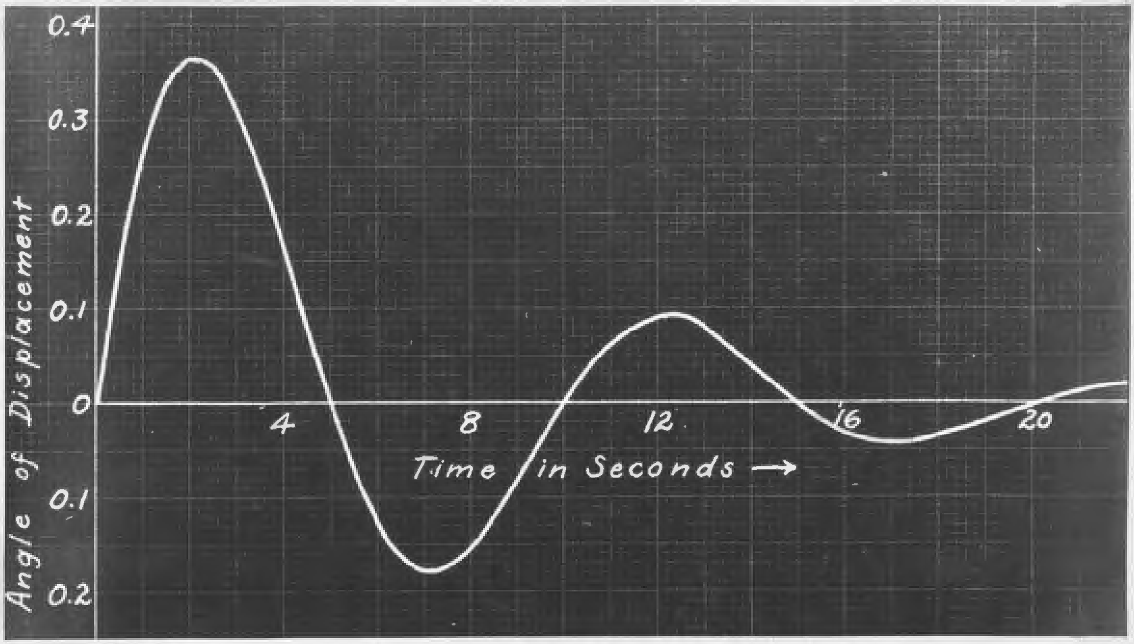
0.4
0.3
0.2
0.1
0
0.1
0.2
Angle of Displacement
4
8
12
16
20
Time in Seconds →

Fig. 2.

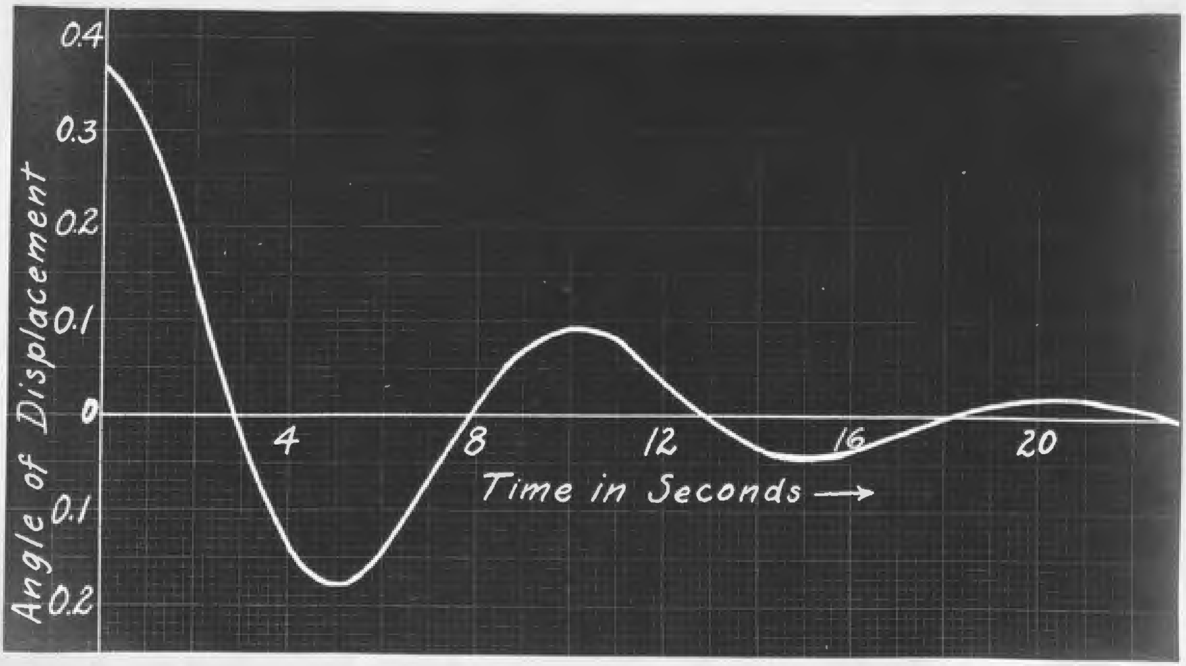
0.4
0.3
0.2
0.1
0
0.1
0.2
Angle of Displacement
4
8
12
16
20
Time in Seconds →

Fig. 3.

The initial conditions may be based on different assumptions. Suppose the coil to be deflected by a steady current through an angle θ_o, in which position it is at rest. At a given instant, $t = 0$, let the circuit be broken, The potential energy in the suspensions will start the coil vibrating. For this case

$$t = 0, \quad \theta = \theta_0, \quad d\theta/dt = 0.$$

With these conditions imposed, eq. (5) becomes

$$\theta = \varepsilon^{-at}(\theta_0 \cos bt + \frac{a}{b}\theta_0 \sin bt). \; . \; .(7)$$

This equation has been plotted in Fig. 3. For the constants the same values were used as in Fig. 2, and in addition θ_0 was taken as 0.366, the maximum displacement which the coil had in eq. (6). The resulting curve is exactly the same as that which resulted from eq. (6), except that the vertical axis has been displaced toward the right a distance equivalent to the time required by the coil to reach its first maximum in Fig. 2.

The further discussion of this case will be based upon eq. (6) since the motion expressed by this equation is that of the ordinary ballistic galvanometer which is not damped to aperiodicity when, for example, a condenser is discharged through it. Furthermore, eq. (6) is simpler to deal with in the analysis that eq. (7), although the latter would lead to exactly the same conclusions.

The Period of the Motion.

By the period of the motion is meant the time required for one complete swing. It may be obtained by finding twice the interval between two successive passages of the coil through the position of rest. To do this we put $\theta = 0$ in eq. (6) and solve for the successive corresponding values of t. The substitution gives

$$\sin bt = 0, \text{ or}$$

$$bt = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots\ldots\ldots, n\pi, \ldots.$$

It follows that

$$t_1 = 0;\ t_2 = \pi/b;\ t_3 = 2\pi/b;\ \ldots\ldots\ldots;$$

$$\text{and}\quad t_n = \frac{(n-1)\pi}{b}\ .$$

The period, T, is given by

$$T = t_{n+2} - t_n = 2\pi/b, \text{ or}$$

$$T = 2\pi \sqrt{\frac{I_o}{q^2 - \frac{f^2}{I_o}}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

The corresponding equation for the period of the undamped motion given by eq. (2) — found by letting f = 0 in the above equation — is

$$\tau = 2\pi \sqrt{\frac{I_o}{q^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

The constants which were used in plotting Fig. 2, when substituted in eq. (8) give T = 10.0 seconds as the period of the coil. Those of Fig. 1, when substituted in eq. (9) give τ = 9.74 seconds as the period of the same coil when there is no damping. Another relation between the periods will be discussed in the next section under Case II.

The Logarithmic Decrement.

The relation between the amplitudes of two consecutive swings is θ_n/θ_{n+1}, where θ_n, θ_{n+1}, ..., are consecutive maximum elongations from the null point. The expressions for the maximum elongations are found by placing $d\theta/dt$ obtained from eq. (6) equal to zero and solving for t_n, t_{n+1}, ..., the instants at which the angular velocity of the coil is zero. The values of t so found are substituted back in the original equation, and the ratio θ_n/θ_{n+1} formed. The value of the ratio so found is numerically equal to $\varepsilon^{\frac{a}{b}\pi}$. This is a constant, hence each amplitude bears a constant relation to the one next following. By proportion

$$\frac{\theta_n}{\theta_{n+1}} = \frac{\theta_n + \theta_{n+1}}{\theta_{n+1} + \theta_{n+2}} = \varepsilon^{\frac{a}{b}\pi}\text{, or,}$$

$$\Lambda = \log_\varepsilon \frac{\theta_n + \theta_{n+1}}{\theta_{n+1} + \theta_{n+2}} = \frac{a}{b}\pi \; . \; . \; . \; . \; . \; . (10)$$

Substituting for a and b their values,

$$\Lambda = \frac{\pi}{\sqrt{\frac{q^2 I_0}{f^2} - 1}} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . (11)$$

The quantity Λ is called the logarithmic decrement of the vibrations.

The experimental method of determining the value of Λ is based on equation (10). If the first swing or amplitude is designated as θ_1, then we may derive from the equation the relation

$$\Lambda = \frac{1}{n}\left(\log \text{nat} \frac{\theta_1}{\theta_{n+1}}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (12)$$

Equations (8), (9) and (11) enable us to find a relation between the periods for damped and undamped vibrations in terms of the logarithmic decrement.

$$\frac{T}{\tau} = \frac{1}{\sqrt{1 - \frac{f^2}{q^2 I_0}}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (13)$$

Equation (11) may be solved for f^2/q^2I_0 in terms of Λ and π, giving

$$\frac{f^2}{q^2 I_0} = \frac{1}{\frac{\pi^2}{\Lambda^2} + 1}.$$

This value substituted in eq. (13) gives

$$T = \tau\sqrt{1 + \frac{\Lambda^2}{\pi^2}}\ ^{1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Equation (14) gives a means of finding the relation between the periods of the same coil for two corresponding values of the logarithmis decrement. Suppose the soil to be vibrating first with the period T_1 and the logarithmic decrement Λ; again with the period T_0 and the logarithmic decrement λ. These values are substituted in (14) and the ratio T_1/T_0 formed. The equation results:

$$T_1 = T_0\sqrt{\frac{\pi^2 + \Lambda^2}{\pi^2 + \lambda^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

This equation shows that in a given coil a comparatively large change is necessary in the logarithmic decrement in order that the period may be appreciably affected; for Λ^2 and λ^2 are small as compared with π^2. In some of the experiments in connection with this work Λ was found never to exceed the value 0.3.

In the curves of Figs. 2 and 3, $\Lambda = 0.708$.

---

[1]Bahrdt, Physikalische Messungsmethoden, p. 112.

Case III. Critically Damped Motion.

If the coefficient of damping is increased sufficiently to make $I_0q^2 = f^2$, the roots of the auxiliary equation are repeated and the solution is

$$\theta = \varepsilon^{-at}(C_1 + C_2t) \ldots \ldots \ldots \ldots (16)$$

To evaluate the constants of integration assume the same initial conditions as in the evaluation of the constants for equations (2) and (6), namely that

$$t = 0, \quad \theta = 0, \quad d\theta/dt = kQ.$$

Equation (16) then becomes

$$\theta = kQt\varepsilon^{-at} \ldots \ldots \ldots \ldots \ldots (17)$$

The equation shows that in this case the motion ceases to be periodic. The coil is said to be critically damped, since it returns to the zero in the shortest possible time without passing beyond. The practical details of the case -- which is an important one on account of the convenience in the use of a galvanometer so damped -- are discussed at length by Jaeger[1] and O.M.Stewart[2].

[1]Ann. d. Phys. 21, 1906, p. 64.
[2]Phys. Rev. 16, 1903, p. 158.

In the evaluation of the constants the initial conditions preceding eq. (7) may be imposed upon eq. (16). That is, $t = 0$, $\theta = \theta_0$, $d\theta/dt = 0$. Then

$$\theta = \theta_0 \varepsilon^{-at}(1+at). \qquad (18)$$

In Fig. 4 is shown the graph of eq. (17); in Fig. 5 that of eq. (18). The constants used are the same as those of the former curves, except that the coefficient of damping was taken as 2.478 instead of 0.545, and that the quantity of electricity discharged through the coil was assumed twice as great. In addition, the θ_0 of Fig. 5 was taken as 0.365, which is equal to the maximum displacement of the coil in Fig. 4. It will be noticed that the curve of Fig. 5 is identical with that of Fig. 4, except that the point corresponding to $t = 0$ in Fig. 5 falls at $t = 1.55$ in Fig. 4, i.e., at the instant when the coil reaches its maximum displacement. A discussion of the comparative maximum displacements in the different cases, and the instants of reaching them, will be given after the next case, that of overdamped motion, has been considered.

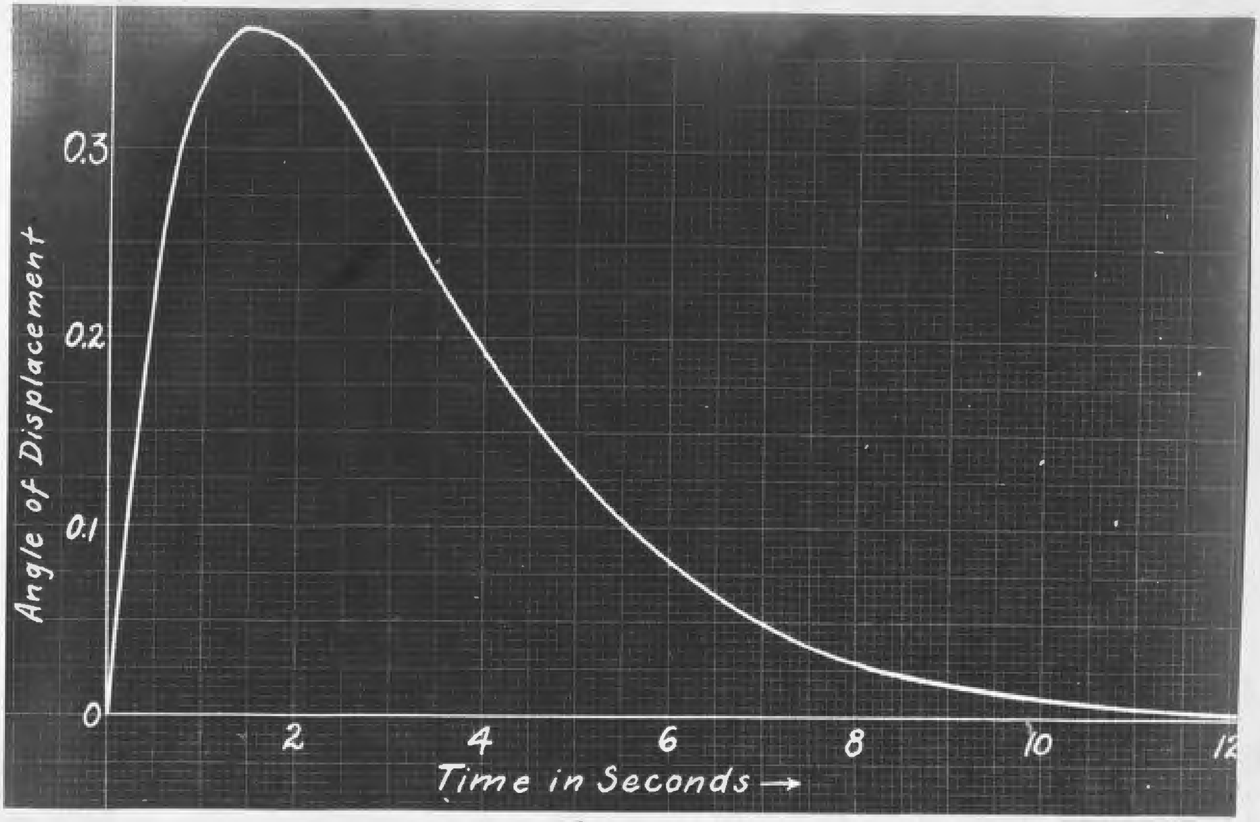
0.3
0.2
0.1
0
Angle of Displacement
2
4
6
8
10
12
Time in Seconds →

Fig. 4.

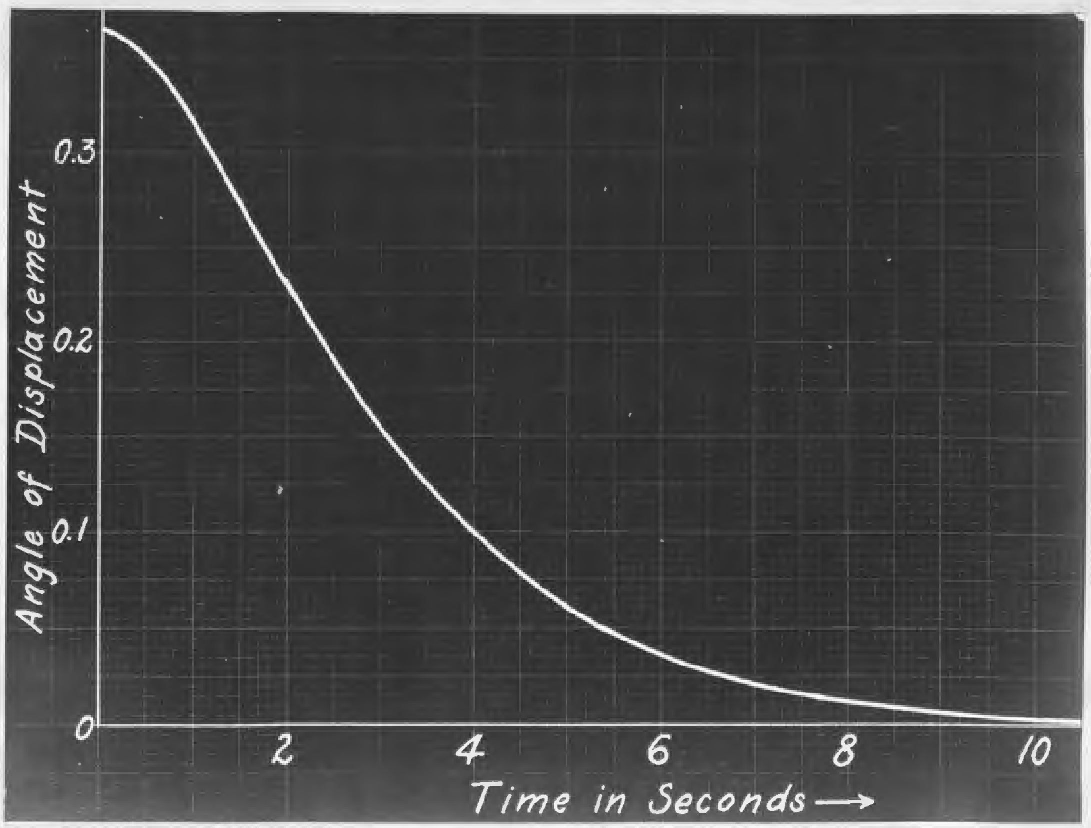
0.3
0.2
0.1
0
Angle of Displacement
2
4
6
8
10
Time in Seconds →

Fig. 5.

Case IV. Overdamped Motion.

This case is relatively unimportant in practice, since the conditions underlying it make the galvanometer practically useless for deflection work and difficult to use in ballistic work. In the theoretical discussion it is of interest, however, especially in its comparison with the other cases which have been mentioned.

To realize the overdamped case, the coefficient of damping has been sufficiently increased to make β real. The solution is given by eq. (4), i.e.,

$$\theta = \varepsilon^{-at}(C_1\varepsilon^{\beta t} + C_2\varepsilon^{-\beta t}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To evaluate the constants, assume the same conditions as in the preceding cases when a quantity of electricity was discharged through the coil. The conditions are

$$t = 0, \quad \theta = 0, \quad d\theta/dt = kQ.$$

Equation (4) becomes

$$\theta = \frac{kQ}{2\beta}\,\varepsilon^{-at}(\varepsilon^{\beta t} - \varepsilon^{-\beta t}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If the initial conditions are taken as representing a return toward the null position of the coil after a steady deflection, the conditions are stated, as before,

$$t = 0, \quad \theta = \theta_0, \quad d\theta/dt = 0.$$

Under these conditions eq. (4) becomes

$$\theta = \varepsilon^{-at}\left[\frac{\theta_0(\beta + a)}{2\beta}\varepsilon^{\beta t} - \frac{2\beta}{\beta + a}\varepsilon^{-\beta t}\right] \;.\;.\;(20)$$

Figs. 6 and 7 represent, respectively, eqs. (19) and (20). The constants have the same values as before, and f is assumed to be 5.76. The curves show the "creeping" of the coil toward the null position after a throw or after a deflection. The damping may be so great that the time required by the coil to return to a small percentage of the maximum displacement may be some minutes.

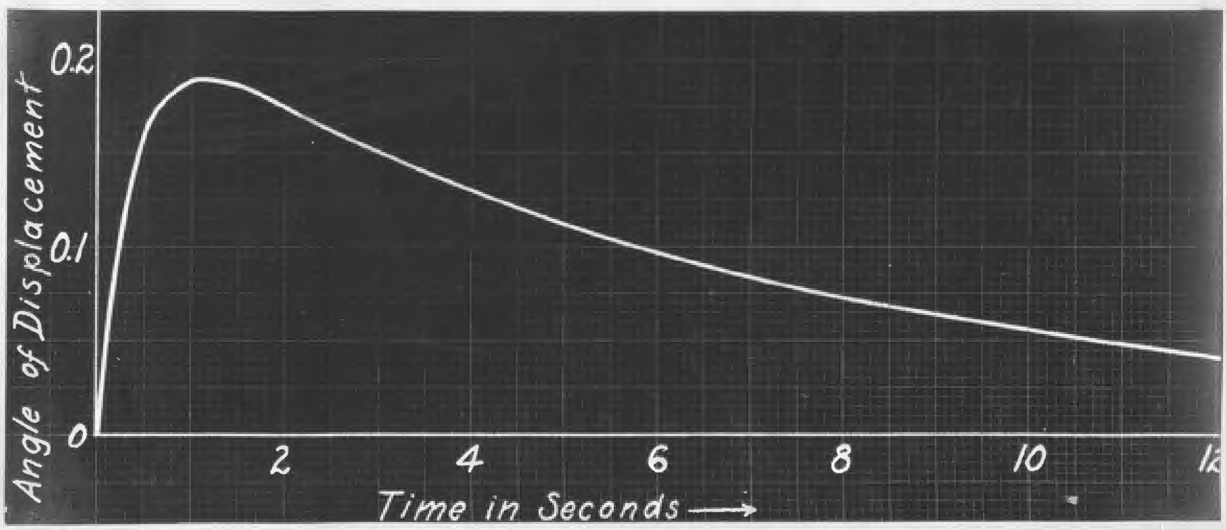
Angle of Displacement
0.2
0.1
0
2
4
6
8
10
Time in Seconds ⟶

Fig. 6.

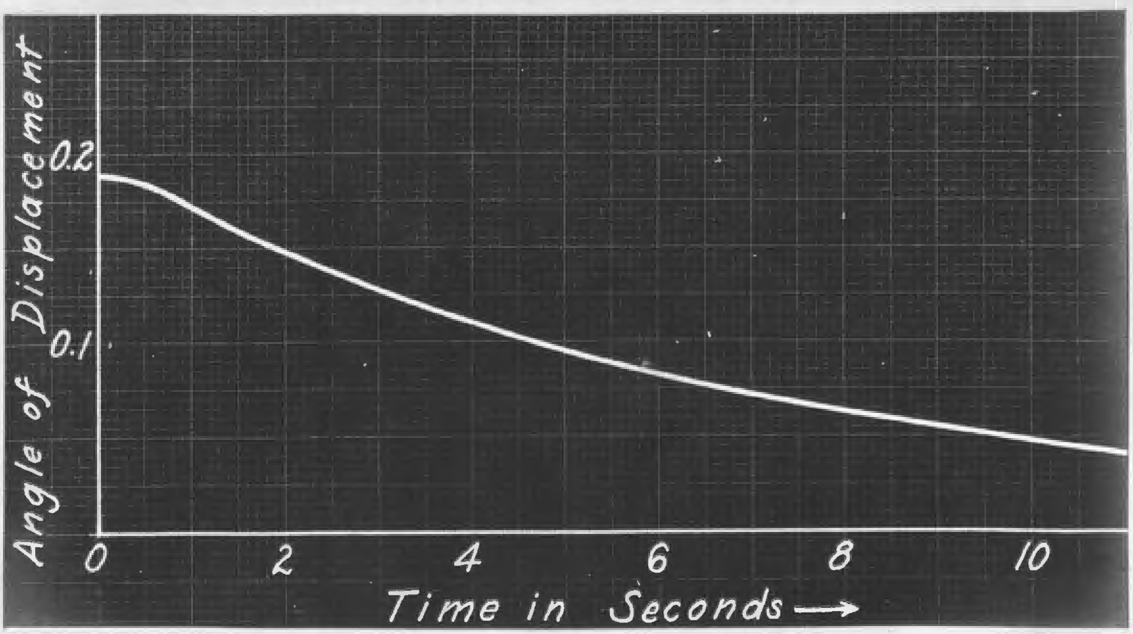
Angle of Displacement
0.2
0.1
0
2
4
6
8
10
Time in Seconds ⟶

Fig. 7.

The Maximum Displacement or Ballistic Throw of the Coil in the Four Cases

In any of these cases the method of finding the throw which a given quantity of electricity will produce is to find an expression for the instant, t_m, at which the angular velocity of the coil, $d\theta/dt$, becomes zero. At this particular instant the angle of displacement begins to diminish. The expression for t_m is then substituted in the equation of motion for the particular case in question. This determines the corresponding maximum value, θ_m, of the angle of displacement. The process applied to the four cases which have been discussed gives the following results:

Case I.

$$t_m = \frac{\pi}{2}\sqrt{\frac{I_0}{q^2}}$$

$$\theta_m = kQ\sqrt{\frac{I_0}{q^2}} = KQ$$

Case II.

$$t_m = \frac{1}{b} \text{ arc tan } \frac{b}{a}$$

$$\theta_m = \varepsilon^{-\frac{a}{b} \text{ arc tan } \frac{b}{a}} \cdot \frac{kQ}{\sqrt{a^2 + b^2}} = K'Q$$

Case III.

$$t_m = \frac{1}{a} = \frac{I_o}{f}$$

$$\theta_m = \frac{kQI_o}{\varepsilon f} = K''Q$$

Case IV.

$$t_m = \frac{1}{2\beta} \text{ log nat } \frac{a+\beta}{a-\beta}$$

$$\theta_m = \frac{kQ}{2\beta}\left[\left(\frac{a+\beta}{a-\beta}\right)^{\frac{\beta-a}{2\beta}} - \left(\frac{a-\beta}{a+\beta}\right)^{\frac{\beta+a}{2\beta}}\right] = K'''Q$$

These equations show the following statements to hold for any one of these kinds of motion, damped or undamped: 1) The time interval from the instant of discharge to the instant of reaching the angle of displacement which represents the ballistic throw is independent of the quantity of electricity discharged.

2) The ballistic throw in any given case is directly proportional to the quantity of electricity discharged through the coil.[1]

We may compare the throws obtained by the same quantity of electricity discharged through the same coil in cases I and II by forming the ratio of the equations given above. Then, remembering that in Case III the condition obtains: $I_0 q^2 = f^2$, we have

$$\frac{\theta_{m1}}{\theta_{m3}} = \frac{kQ\sqrt{\frac{I_0}{q^2}}}{kQ\sqrt{\frac{I_0}{q^2}}} \cdot \frac{1}{\varepsilon^{-1}} = \varepsilon \; . \; . \; . \; . \; . \; . \; . \; . \; (21)$$

This equation shows that when the galvanometer is undamped, the throw which a given quantity of electricity will produce is about 2.7 times as great as the throw which the same quantity will produce in the same galvanometer when critically damped.

---

[1]This statement as well as the relation between the throws which follows have been given by O.M.Stewart. See Phys. Rev. 16, 1903, p. 160.

B) APPLICATION OF A DAMPED COIL TO THE MEASUREMENT OF MAGNETIC FIELDS

Two methods are commonly used in the determination of field intensities.

The first of these, the ballistic method, depends upon the fact that when lines of magnetic force are cut by a conductor which is part of a closed circuit, a quantity of electricity is induced in that circuit proportional to the number of lines cut. A test coil is connected in series with a ballistic galvanometer. If now the test coil, which was placed perpendicular to the direction of the lines of force, is suddenly withdrawn from the field, a throw is pro duced on the galvanometer. The throw may be reduced to corresponding field intensity if the constant[1] of the galvanometer is known, as well as the area and number of turns of the test coil, and the total resistance of the galvanometer circuit. The constant may be

[1]A. Zeleny, Phys. Rev. 23, 1906, p. 408 et seq.

found either by means of a condenser of known capacity charged to a known potential, or by means of a mutual inductance coil the coefficient of which is known or calculable. To produce the throw, the test coil may be rotated suddenly through 180°, like an earth inductor. In this case the lines are cut twice, so that the throw is twice as great as when the coil is withdrawn from the field. In any case, the construction of the test coil and the determination of its dimensions, together with the determination of the constants of the other apparatus used, is rather a laborious process.

The other method, that of the bismuth spiral,[2] is applicable only to very strong fields. It consists in the determination of the resistance of the spiral when placed in a magnetic field. The corresponding field intensity may be obtained from the calibration curve which is furnished with the instrument. By this method results are obtainable which may, in general,

---

[1]Starke, Experimentelle Elektrizitätslehre, (2nd ed., 1910) p. 251.

be relied upon for an accuracy of five percent. The bismuth spiral has the disadvantage of having to be used near the temperature at which the calibration curve was obtained, since the resistance temperature coefficient for bismuth is nearly as great as that for copper. The method has the advantage of being simpler in application than the ballistic method, since it requires only a Wheatstone's bridge, together with the accessories.

In this paper a method is to be presented which has certain advantages over the methods just outlined. It requires no separate galvanometer, and no source of current or electromotive force as the other methods do. The apparatus necessary for measurements by this method consists of a galvanometer coil, or other suspended coil provided with a mirror, so that readings of the angular positions of the coil may be taken; a resistance of known value, or a variable resistance which may be connected in series with the coil, and a stop watch. The results are unappreciably influenced by temperature changes, and no calibration curve is necessary to obtain them. Finally, the observed values need not be determined with great accuracy, since in the reduction they enter under the radical sign. The result has therefore twice the accuracy of the observations. The application of the method will be discussed at greater length after the theory has been developed.

Theory of the Method

In addition to the natural damping of a coil swinging in a magnetic field -- damping due to air friction, eddy currents and viscosity of the suspensions -- there may be artificial damping by means of light vanes attached to the galvanometer coil, or by means of a short-circuited copper rectangle attached to the coil, or by means of a resistance to which the coil may be connected. The last method mentioned is the basis of the theory here to be given. When a coil is connected with a resistance, the two forming a closed circuit, damping takes place on account of the fact that the system acts like a dynamo. The swinging coil, cutting the lines of force of the field, is the source of an electromotive force which causes a current to flow in the circuit. The source of the energy producing the current is the work which was originally done upon the coil to set it in motion. If the motion of the coil is periodic a partial interchange of energy -- kinetic to potential and vice versa -- takes place with each swing. Some of the energy is changed to heat, and the amount of energy so changed determines the amount of the damping, since the heat energy is derived from the electrical energy produced by the swings. Consequently, the smaller the total resistance in the circuit, the greater will be the current induced, and therefore the greater will be the amount of energy changed from the kinetic form to that of heat. In other words, the smaller the resistance in the circuit, the greater will be the coefficient of damping and with it the logarithmic decrement.

In addition to the notation already introduced in Section A of this paper, the following designations will be used:

I = current induced in the swinging coil circuit at any instant;

T_0 = period of the coil on open circuit;

T_1 = period of the coil on closed circuit;

R_1 = total resistance in the circuit, to which corresponds the period T_1;

R_0 = total resistance in the circuit to make the coil just aperiodic;

λ = logarithmic decrement on open circuit;

Λ = logarithmic decrement on closed circuit.

In these derivations the angle of displacement θ will be assumed so small that cos θ remains practically unity. This assumption is permissible since the angle may become 6° before the difference between its cosine and unity exceeds 0.5%. With the scale and telescope at a distance of 50 cm. this angle is equivalent to a deflection of about 10 cm. The reason for making the assumption is because in the theory it is supposed that at any instant the coil cuts the lines of force perpendicularly. When the coil swings about an iron core, as in the ordinary galvanometer, the field is approximately radial for angles greater than 6°; in this case the assumption is even more permissible.

At any instant t let the coil have the angular velocity $d\theta/dt$. The rate of cutting lines of force at this instant is

$$N = nlrH \frac{d\theta}{dt} \text{ per second.}$$

From the definition, N is numerically equal to the electromotive force induced in the coil, measured in electromagnetic c.g.s. units. By Ohm's law, the electromotive force produces in the circuit of which the moving coil is a part a current which is given by the equation

$$I = \frac{nlrH \cdot \frac{d\theta}{dt}}{R_1}.$$

The moment of the force opposing the motion of the of the coil due to this induced current only is, at the instant t, equal to nlrHI; therefore the damping moment due to this induced current (leaving out of consideration for the present all other causes of damping) is

$$\frac{n^2l^2r^2H^2 \frac{d\theta}{dt}}{R_1}.$$

In eq. (3), the original differantial equation, the moment of the damping force is the angular velocity of the coil multiplied by the constant 2f; in other words, the assumption was made that the damping moment is proportional to the angular velocity. The expression given at the bottom of the preceding page shows that the fraction of the damping moment due to induced currents -- which includes that due to eddy currents -- is actually proportional to the angular velocity of the coil. The assumption originally made, therefore, need cover only that part of the damping which results from air friction and viscosity of the suspensions. We might, after these considerations, equate the coefficients of the term $d\theta/dt$ in the original equation and in the expression for the damping moment which was above given; that is, we might say: $2f = (nlrH)^2/R_1$, provided there were no damping other than that due to the induced current in the circuit. In the actual case, however, there are other factors which go to make up the coefficient of damping.

Let m represent that part of 2f which is <u>not</u> due to the induced current, i.e., which <u>is</u> due to eddy currents and the other causes mentioned. This definition of m shows that quantity to be simply the damping coefficient when the coil swings on open circuit. We may therefore write:

$$f = \frac{n^2 l^2 r^2 H^2}{2R_1} + \frac{m}{2} \quad \ldots\ldots\ldots\ldots (22)$$

A value may now be found for f on closed circuit and a similar value for m/2, which is nothing more than f on open circuit. By eq. (11)

$$q^2 I_0 = \frac{f^2(\pi^2 + \Lambda^2)}{\Lambda^2} \quad \ldots\ldots\ldots\ldots (23)$$

and by eq. (8)

$$T_1 = 2\pi \sqrt{\frac{I_0}{q^2 I_0 - f^2}} \quad \ldots\ldots\ldots\ldots (24)$$

Substituting (23) in (24) and solving for f,

$$f = \frac{2I_0\Lambda}{T_1} \quad \ldots\ldots\ldots\ldots (25)$$

Similarly, when the coil is swinging on open circuit,

$$\frac{m}{2} = \frac{2I_0\lambda}{T_0} \quad \ldots\ldots\ldots\ldots (26)$$

Equation (15) shows that except for large differences in Λ and $\dot{\lambda}$ there is no appreciable difference between T_1 and T_0. Furthermore, it will be seen in the final equation that the period of the coil is one of the observed quantities which will enter under the radical sign, and therefore its accuracy need be only half as great as the desired accuracy of the result. It is therefore permissible to replace T_1 in (25) by T_0. The value of f given by (25) and the value of m/2 given by (26) may now be substituted in eq. (22), and the resulting expression solved for H. The equation follows:

$$H = \frac{1}{nlr}\sqrt{\frac{4I_0R_1}{T_0}(\Lambda - \lambda)} \quad . \; . \; . \; . \; . \; . \; .(27)$$

Each of the quantities in the above equation is expressed in the electromagnetic c.g.s. system of units; if, therefore, R_1 be measure in ohms, it must be multiplied by the factor 10^9. Another reason for having replaced T_1 by T_0 may now be mentioned: T_0 is more easily found experimentally, since it is the period of the coil on open circuit. For this reason more swings may be obtained at a single observation than when the coil is damped more strongly.

The application of the formula is not difficult. T_0 has already been mentioned; it is obtained directly by means of a stop watch. Λ and λ are found by eq. (12). The other quantities involved, which are constant in all measurements, may be determined once for all: n is either given by the manufacturer of the coil, or determined when the coil is constructed; l and r may be conveniently measured by means of a microscope micrometer; and I_0 is found by comparing the period of the coil with the period of a body of which the moment of inertia is known, each in turn being suspended by means of the same fiber.

Since another equation is to be developed which is based upon critical damping, the method above given will hereafter be referred to as "the periodic method."

By a very similar process of reasoning it is possible to find another expression which may be applied in the determination of field strengths, making use of the criterion for critical damping, which may be stated

$$2f = 2\sqrt{q^2 I_0},$$

the quantities entering into the equation being defined as before. The coefficient of damping, 2f, again consists of a) the part due to the change of the kinetic energy of the coil to electrical energy; i.e., the part due to the induced current; and b) the part due to eddy currents, viscosity of the fiber and air friction. Neither part of the damping coefficient is dependent upon the periodicity or aperiodicity of the coil, but both are proportional to the angular velocity of the coil, no matter what the character of the motion at any instant may be. The second part of the damping coefficient may, as before, be represented by the quantity m.

When the motion is just aperiodic, then, the equation of damping may be expressed in the form[1]

$$\frac{n^2l^2r^2H^2}{R_0} = 2\sqrt{q^2I_0} - m. \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

By eq. (26)

$$m = \frac{4I_0\lambda}{T_0},$$

and since q^2I_0 is constant for any coil, whether the coil is swinging on open or on closed circuit,

$$q^2I_0 = \frac{4I_0{}^2(\pi^2 + \lambda^2)}{T_0{}^2} \; .$$

These values are substituted in eq. (28) and the resulting expression solved for H. The result is

$$H = \frac{1}{nlr}\sqrt{\frac{4I_0R_0}{T_0}\left(\sqrt{\pi^2 + \lambda^2} - \lambda\right)}. \quad . \quad . \quad . (29)$$

Since λ^2 is very small as compared with π^2, we may write

$$H = \frac{1}{nlr}\sqrt{\frac{4I_0R_0}{T_0}(\pi - \lambda)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

---

[1]W. P. White, Phys. Rev. 23, 1906, p. 389.

Equations (30) and (27) much resemble each other. The only difference in their application to field measurements is that in (30) a determination of R_0 replaces the readings required for obtaining the value of Λ in eq. (27). It is interesting to note that there is a correspondence between the quantities R_1 and Λ in (27) and the quantities R_0 and π in (30). The significance of this fact is that R_0 is the limiting value of R_1 for which the coil becomes just aperiodic; and that the limiting value of the logarithmic decrement when the coil is critically damped is, curiously, the constant π. It is a little difficult, from a consideration of the definition of the logarithmic decrement, to see the interpretation. It is hoped that another method may be found to show conclusively that the value of the logarithmic decrement, as the coil approaches critical damping, becomes more and more nearly equal to the constant π.

Referring again to eq. (30), since R_0 is the resistance which will make the coil just aperiodic, there is a minimum field strength in which this equation may be applied. The limits within which (27) may be used are very wide. With the same coil we are able to measure fields of a few lines up to fields of thousands of lines. By changing R_1 we may make any number of independent determinations of the same field strength. Or, if the field intensity is great enough to make the coil aperiodic on closed circuit, equation (30) may be used to check (27), since the readings for both equations are obtainable with the same setting of the instruments.

If no magnetic impurities[1] were present within the coil, T_0 in either equation would remain constant for all field strengths, which would simplify the application of the formula. The variation in T_0 with field intensity will be discussed in another section of this paper.

---

[1]Stansfield, Phil. Mag. 46, (5), 1898, p. 67; Diesselhorst, Zschr. für Instrumentenkunde 31, 1911, p. 278; A. Zeleny, Phys. Rev. 32, 1911, p. 297.

There is another advantage in the damped coil method over the ballistic method of measuring magnetic fields. It is almost impossible, if not quite, to measure by the ballistic method the field intensity of the ordinary moving-coil galvanometer having an iron core between its poles. By means of specially designed apparatus Zeleny and Hovda[1] measured the total number of lines of force in such a field, but on account of the distribution of the lines no accurate value of H for the region occupied by the coil can be given. On the other hand, the damped coil method presents no difficulties to such a determination. The manipulation is the same whether or not there is an iron core in the field. By this method, then, accurate information concerning the change produced by the introduction of an iron core into a magnetic field may be obtained.

[1]Phys. Rev. 28, 1909, p. 283.

An Equation for Finding the Resistance of a Damping Rectangle to Make the Coil Just Aperiodic

When the field strength of a galvanometer has been measured by one of the above methods, eq. (30) enables us to find the resistance and from it the size of the wire which must be used in an auxiliary damping rectangle in order that the galvanometer coil to which the rectangle is attached may be critically damped, or very nearly so. The equation is solved for R_0 and n is placed equal to unity, since there is a single turn of wire. This gives

$$R_0 = \frac{(Hlr)^2 \, T_0}{4I_0(\pi - \lambda)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (31)$$

Both period and moment of inertia are slightly altered by adding a mass equal to that of the rectangle and, of course, similarly distributed, to the coil. But I_0 varies as T^2, nearly, and the change in either quantity due to the increased mass is small; and finally, the size of the wire to be used in the rectangle depends, to some extent, upon the standard wire sizes. Ordinarily, therefore, the difference may be neglected and the resistance found by eq. (31) reduced to the corresponding diameter of wire by means of the dimensions of the coil. It should be noted that the R_0 obtained from this equation is in electromagnetic c.g.s. units and should be reduced to ohms or microhms by the proper reduction factor.

A Damping Method for Finding the Resistance of the Moving Coil in a Galvanometer

A method has been given by A. Zeleny[1] for finding the resistance of a galvanometer coil by letting it swing from a known deflection and observing the distances it moves beyond the null point when connected in series with first one and then another known resistance. Reduction of the values so obtained by means of a given curve gives a new pair of values which may be substituted

[1] A. Zeleny, Phys. Rev. 23, 1906, p. 420.

in a given equation and the resistance of the coil solved for. A modification of the same method, employing a direct observation of the respective logarithmic decrements for two known resistances in series with the coil, is based upon eq. (27).

Let R_c = resistance of the coil;

R' = resistance in series with the coil giving log. dec. Λ';

R'' = resistance in series with the coil giving log. dec. Λ''.

These values are substituted in eq. (27) and R_c found in terms of the known quantities. The formula is

$$R_c = \frac{R''\Lambda'' - R'\Lambda' - (R'' - R')\lambda}{\Lambda' - \Lambda''} \quad . \ . \ . \ (32)$$

If R'' is taken equal to $2R'$, (32) becomes

$$R_c = \frac{R'(2\Lambda'' - \Lambda' - \lambda)}{\Lambda' - \Lambda''} \quad . \ . \ . \ . \ . \ . \ (32a)$$

Experimental Tests of Periodic and Aperiodic Damped Coil Methods

For the purpose of verifying equations (27) and (30) it was necessary to use apparatus by means of which known field values could be compared with the values obtained by the use of the formulas. It was desirable to have the magnetic field uniform over an area somewhat greater than that of the galvanometer coil to be used, so that the field intensity might actually be a certain number of lines in every square centimeter over this area. To obtain such a field which could at the same time be varied in intensity from zero to about fifteen hundred lines, an electromagnet was used, to the adjustable poles of which were attached flat parallel pole pieces of cast iron, of dimensions 11.9 x 7.9 x 1.65 cm., with an air gap of 3.85 cm. between them.

Small magnetizing currents were measured on a milliammeter of range 0-200. This was connected in series with an ammeter of range 0-5, the milliammeter being short circuited when current values higher than 0.2 ampere were used. The ammeters were connected in series with a storage battery, a rheostat and a reversing switch. The electromagnet was connected to the other two binding posts of the reversing switch, so that the current might be sent through the turns of the magnet in either direction. This arrangement also made it possible to subject the iron of the magnet to any number of cycles of hysteresis, so that any point on the magnetization curve and therefore a definite field strength could be obtained.

In the later observations with the damped coil it was impracticable to make a direct comparison of field intensities each time a determination of one particular value of the intensity was made. For this reason a magnetization curve was first obtained by the ballistic method, using a "snap coil" on the principle of the earth inductor for obtaining the ballistic throws for the different field values. The test coil was rectangular, wound on a flat plate of ebonite having a groove cut around its edge; its mean length was 4.950 cm., its mean width 1.830 cm., both being measured on a microscope micrometer. On the plate were wound 30 turns of No. 40 S.S.C. copper wire. The plate was mounted in a frame which could be supported between the poles of the magnet. By means of a rubber band, the coil could be made to "snap" through 180° in a small fraction of a second. Throws from the test coil were compared with those from a mutual inductance coil having a coefficient of 0.017653 ± 5 henry; currents in the primary of this coil were measured with a Wolff potentiometer and a Wolff standard resistance. The magnetization curve is shown in Fig. 8. From this curve (drawn to a much larger scale) were obtained the values of H with which the intensities as measured by the damped coil methods were compared. The reliability of the values obtained from the curve depends to a great extent upon the accuracy of the ammeters and the observational errors in reading the ammeters. An uncertainty arises from the fact that the "constant" of a ballistic galvanometer varies somewhat with the magnitude of the throw, and the character of this variation is not known; the "constants" as determined by three different methods also show a variation among themselves. The probable errors of the values taken from the curve, corresponding to the readings of the magnetizing currents, are for the reasons mentioned, not less than 0.4%, may be as great as 0.6%, and for very low field values may be much larger. As will be seen from the curve, zero field between the poles of the magnet was obtained by passing a "demagnetizing" current through the turns of the electromagnet after it had been subjected to several reversals of magnetization. This current had a value of 0.0922 amp., as determined from a large number of observations.

The quantities in equations (27) and (30) which are constant throughout the determinations were first measured. The measurements were made with a Leeds & Northrup galvanometer coil . The resistance of the coil was measured by means of a Wheatstone's bridge, the value found being 121.7 ohms. The number of turns given by the manufacturers is 395, but since one terminal is at the top and the other at the bottom of the coil, this number is in error by plus or minus five-tenths of a turn. The dimensions of the coil were measured on a micrometer microscope, an average of twelve readings for each dimension giving as its mean length 4.853 cm., and as its mean width 1.696 cm. The moment of inertia of the coil was determined by comparing its period with that of a brass disk, each in turn suspended from the same fiber -- a 1.5-mil phosphor bronze strip. The brass disk was accurately turned. Its mean diameter was found to be 2.000 cm., its mean thickness 0.2094 cm. On the axis of the disk were mounted, one on each side, two small terminals such as are used on the L. & N. galvanometer coils. The total moment of inertia of disk and connectors was found to be 2.790 c.g.s. units. The period of the disk was 34.25 seconds, that of the coil 36.68 seconds.

The resulting value for the moment of inertia of the coil is 3.200 c.g.s. units.

During the experiments the coil was suspended in a brass case having a glass cover, so that the reflected image of a scale could be obtained in a reading telescope from the mirror mounted on the coil. The width of the case was the same as that of the frame in which the test coil was mounted, so that the width of the air gap between the poles of the magnet was the same as during the determination of the magnetization curve. The moving coil was suspended as nearly as possible in the position for which the ballistic throws had been obtained with the test coil. The suspensions were of phospher bronze: 3-mil upper and 1.5-mil spiral lower, the lengths being nearly the same as those in the H and P types of galvanometers of the L. & N. Co. The scale was circular, of 50.0 cm radius, and mounted at a distance of 50.0 cm. from the mirror.

Readings for the period were obtained by means a stop watch which had been rated by comparison with the laboratory clock. For both periodic and aperiodic methods readings were first taken for the logarithmic decrement on open circuit; then a resistance was connected in series with the coil, and the readings for the logarithmic decrement on closed circuit for eq. (27) taken. For all logarithmic decrements an average of the values found from fifteen or more vibrations of the coil was used. Finally, when the field was sufficiently strong, R_0 for eq. (30) was found.

In order to displace the coil from its null position its terminals were connected to the "Galv." posts of a Zeleny key which was used with a condenser of 0.5 microfarad capacity and a battery of 4 volts. By tapping the key once for strong fields and several times for low fields a displacement of the desired magnitude could be obtained. This displacement was kept, as far as possible, within the angle of 6° mentioned in the theoretical development of the method.

Results of Measurements by the Periodic Method

The periodic method of eq. (27) was used for measuring fields ranging in strength from a few lines up to about fifteen hundred. As has been pointed out before, this method is somewhat more useful than the aperiodic method of eq. (30) for the reason that it may be applied to fields below which the coil is just aperiodic on short circuit. Table I is a review of the results together with the observations in a series of readings made for the periodic method. In Fig. 8 are plotted the corresponding points.

For the higher intensities the greatest difference between the values as determined by the periodic and ballistic methods is 1.6%. Since the results obtained by the ballistic method is subject to error to even a greater extent than those by the other method, the two columns of values do not show the amounts by which the results of the new method differ from true values; but rather, they show that the method of eq. (27) com-

Table 1.

Magnetizing Current	T_o	λ	R_1	Λ	H (by eq. 27)	H (ballistic method)
0.0998	7.948	.01102	121.7	.01170	3.404	4.4 (abt)
0.1199	7.945	.01101	121.7	.02242	14.54	16.5 (")
0.1885	7.967	.01128	121.7	.1872	57.0	57.0
0.2630	7.974	.01190	621.7	.1277	104.5	105.7
0.3320	7.948	.01278	1,122	.1399	147.4	148.5
0.4140	7.890	.01413	1,622	.1747	199.8	199.0
0.5660	7.478	.01742	3,500	.1708	294.7	295.0
0.9830	6.913	.03060	10,000	.2026	548.5	542
1.421	6.428	.04614	20,000	.2079	780	770
1.854	6.128	.06196	40,000	.1800	966	964
2.350	5.891	.07769	50,000	.2025	1133	1143
3.500	5.630	.09800	100,000	.1892	1400	1405
4.330	5.503	.1068	100,000	.2129	1528	1503

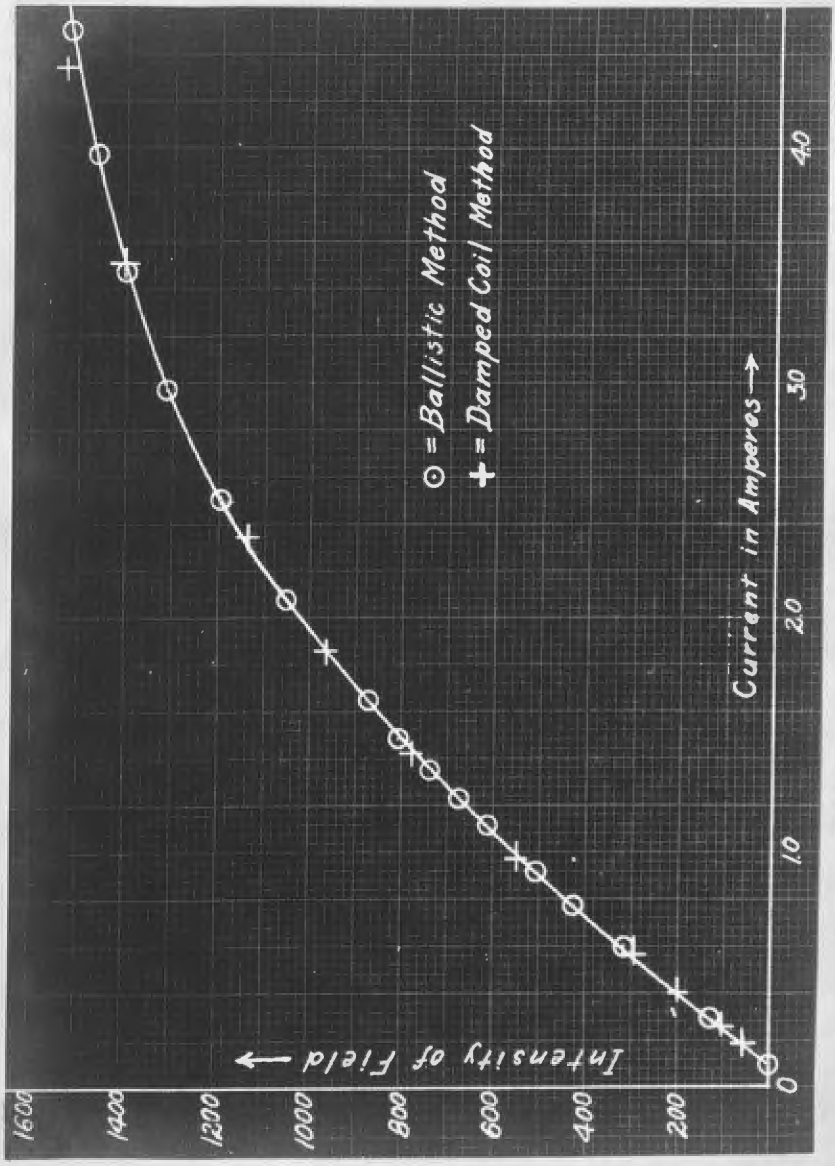
⊙ = Ballistic Method
+ = Damped Coil Method
Current in Amperes →
Intensity of Field →
0
10
20
30
40
200
400
600
800
1000
1200
1400
1600

Fig. 8.

pares favorably with the ballistic method when applied to measurements of the same quantities. For field strengths below thirty lines the ballistic method, with the size of the test coil used, is not dependable, on account of the very small throws which these fields gave in the determination of the magnetization curve. The damped coil method here presented may therefore be relied upon for an accuracy of 1%, and perhaps even a fraction of 1%.[1] Individual determinations by this method with different values of R, agree within 0.1%, but this is not entirely indicative of the accuracy of the measurement, since the same small constant error may affect all the determinations.

Application of the Periodic Method to a Measurement of the Horizontal Component of the Earth's Field.

As a matter of interest more than in the hope

---

[1] W. P. White, Phys. Rev. 23, 1906, p. 390, gives an equation for the determination of field strength which may be used for determining galvanometer fields to an accuracy of 10%, according to the author. The equation is rather difficult of application.

of obtaining quantitive results of value, the method was applied to the measurement of the horizontal intensity of the earth's magnetic field. The coil was suspended in its case with its plane approximately parallel to the direction of the magnetic lines of the earth's field. All of the variable quantities were determined with great care, and two independent sets of readings taken. In the first set the value of $\Lambda - \lambda$, as obtained from observations upon fifty complete vibrations, was found to be 2.53×10^{-6}; in the second set the same quantity had the value 3.27×10^{-6}. The periods were, respectively, 7.810 and 7.818 seconds, found in each case by timing 125 complete vibrations. R_1 in both sets had the value 121.7 ohms, or 1.217×10^{11} c.g.s. units of resistance. The resulting values of H are 0.218 and 0.247 lines per square centimeter, while the value as found by means of an earth inductor, is 0.161 lines. Since the coil has a total area of only 10 cm^2., the electromotive force induced in it when moving at its maximum angular velocity is extremely small, hence the damping due to the induced current when the coil is short circuited is very minute. The quantity ($\Lambda - \lambda$) therefore is the difference between two quantities which are very small and almost equal in value, so that the probable error in their difference must become very large. The results, of course, are of little value. They simply indicate that at present the method as applied to very weak fields is only qualitative, but that possibly it may be developed into one of greater precision. More work will be done upon this particular application of the periodic method.

Results of Measurements by the Aperiodic Method

The aperiodic method of eq. (30) gives results which do not agree nearly so well with the results obtained by the ballistic method as those of the periodic method. The results in all cases are higher than those by the other two methods, and the deviation increases for increasing fields. At 300 lines it is about 3.6%, and at 1500 lines 7.2%. The indication is that magnetic impurities in the coil, to which reference has already been made in this paper, are the cause of the difference. Just in what manner the impurities affect the quantities which enter into the equation has not been worked out. More work will be done upon this particular point in the hope of finding why such variation should exist.

A calibration curve could, of course, be drawn, and the values obtained by substitution in the formula corrected from the curve. For any particular coil this might even be practicable, on account of the readiness with which the observations can be made. For galvanometer fields, the method may be depended upon for an accuracy of 5% without such a calibration. In view of the fact, however, that the periodic method gives such good results, it is preferable to use that method.

An investigation of the disagreement in results between the two methods may possibly throw some light upon the question of effect of magnetic impurities in galvanometer coils.

C) PECULIARITIES IN THE BEHAVIOR OF THE MOVING COIL

1) Period of the Coil on Open Circuit.

During the experimental work upon the damped coil methods of measuring magnetic fields, a number of peculiarities in connection with the motion and damping of the coil were noticed. The first of these appeared in the determination of periods of the coil with varying fields.

It was pointed out by Stansfield[1] as early as 1898 that the apparent set in the suspension fiber immediately following a deflection of the moving coil in a given direction was really due to magnetic impurities in the materials of which the coil was constructed. The suggestion seems not to have been generally accepted, though others mentioned the same thing[2] several years later. In 1911 a paper appeared[3] which

---

[1] Phil. Mag. 46, 1898, p. 67.
[2] A. Zeleny, Phys. Rev. 23, 1906, p. 399; White, Phys. Rev. 23, 1906, p. 382; Diesselhorst, Zschr. f. Instrumentenkunde, 31, 1911, p. 247.
[3] A. Zeleny, Phys. Rev. 32, 1911, p. 297.

treated of zero shift and deflection hysteresis in moving-coil galvanometers, and which showed that these peculiarities were due to the magnetic impurities which exist in all galvanometer coils.

In their respective papers White and Zeleny call attention to the fact that the period of the coil diminishes as the strength of the field increases. The nature of this variation has been investigated in some detail in the present work.

It was observed that as the field was increased from zero to higher values, the period of the coil at first increased. The maximum value of the increase came at about ninety lines per square centimeter; then followed a diminution, and at about 180 lines the period had practically the same value as at zero field. From this point on the diminution in period continues quite rapidly, then more slowly. The portion of the curve above 180 lines suggests a saturation curve, and therefore indicates that the change is due to magnetic impurities.

In order to ascertain more readily just what the nature of the change in period with increasing weak fields might be, a chronograph was used to measure accurately the periods. By means of a telegraph key the instant of transit of the scale zero across the vertical hair in the telescope could be accurately recorded, a number of complete swings counted, and another record made of the transit of the zero at the completion of the last swing. The set of values of Table 2 was obtained, to which corresponds Curve I in Fig. 9.

The interpretation of Curve I is found in the magnetic impurities of the coil and the method of procedure which was followed in the experiment. It has already been explained that in order to obtain zero field between the poles of the electromagnet, the latter was subjected to several reversals of magnetization, and finally a "demagnetizing current" of certain value was caused to flow through the turns of the magnet in the direction opposite to that which produced the previous maximum magnetization. During this whole process the coil was suspended between the poles of the magnet; therefore the impurities in the coil were subjected to exactly the same process of magnetization as the iron of the magnet. The result was that at zero field the impurities were magnetized in the opposite direction to that which the increasing field would have; i.e., the "N-pole" of the magnetic impurities is nearest the N pole of the magnet, and the same is true of the other side of the coil. Now, as the field is increased, there is repulsion between like poles. The magnetic moment of the impurities in the coil with the field is acting in the direction opposite that of the torsional moment of the fiber at every position of the coil. In general, the period is inversely proportional to the square root of the torsional moment; if, therefore, the resultant moment is diminished, the period will be increased. After a certain field strength has been reached the impurities will be magnetized in the same direction as the

Table 2

Intensity of Field	Period
0	7.779
24	7.801
51	7.820
81	7.826
113	7.824
151	7.802
205	7.748
311	7.561
412	7.342
523	7.072
635	6.610
806	6.278
1050	5.937
1193	5.746
1510	5.412

Table 3

Intensity of Field	Period
0	7.785
27	7.765
52	7.729
81	7.698
115	7.643
154	7.594
204	7.501
313	7.274
421	7.055
526	6.825
645	6.568
817	6.274
1049	5.885
1204	5.692
1515	5.411

field; and from this point on the diminution of period will be continuous.

To show that this explanation is acceptable, the following method was used. The coil was strongly magnetized in a given direction and removed from the field. The magnet was then put through an half-cycle of hysteresis, and the coil replaced. Under these conditions the magnetization of the impurities and the increasing field will have the same direction. It is then to be expected that as the field is increased from zero, the period will immediately begin to decrease without first rising to a maximum; for the resultant magnetic moment is added to, instead of subtracted from, the torsional moment from the beginning. With this order of procedure, the set of values of Table 3 was obtained, to which corresponds Curve II of Fig. 9. The curves of this figure show the possibility of obtaining a complete "hysteresis curve" between period and field. They show conclusively that the change in period is due to the magnetic impurities.

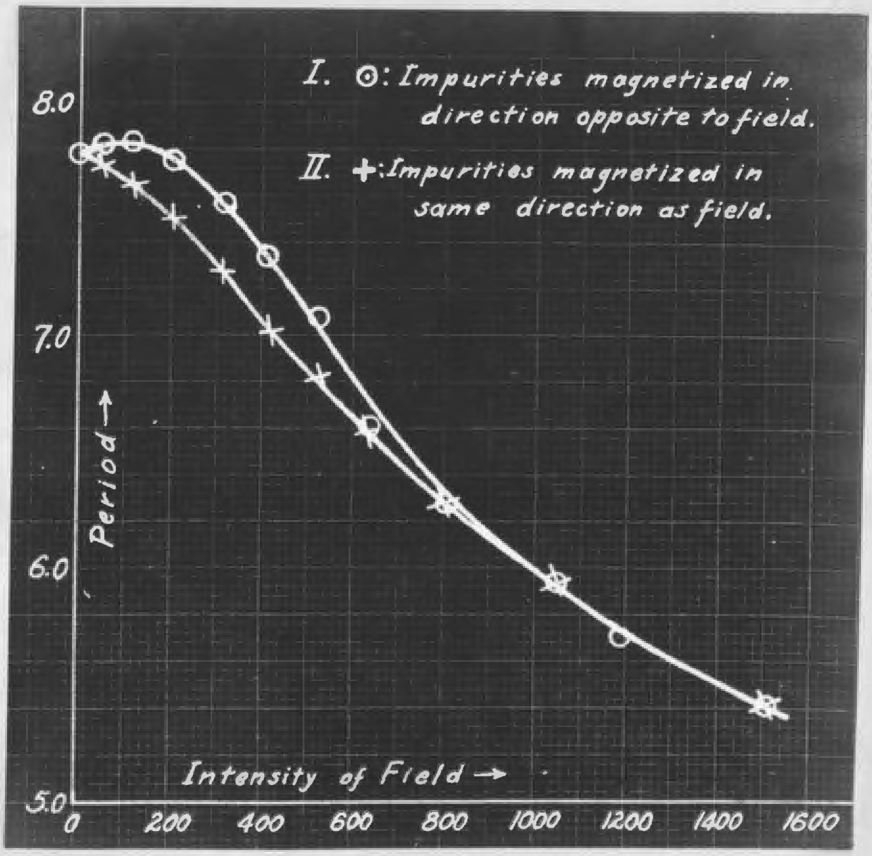
I. ⊙: Impurities magnetized in direction opposite to field.
II. +: Impurities magnetized in same direction as field.
8.0
7.0
6.0
5.0
Period →
Intensity of Field →
0
200
400
600
800
1000
1200
1400
1600

Fig. 9.

2) Logarithmic Decrement of Coil on Open Circuit

Table I shows that as the field increases, the logarithmic decrement on open circuit also increases. To study the manner of variation more thoroughly, a continuous set of readings was taken for fields between zero and 1500, both at atmospheric pressure and at a pressure of about 0.5 mm. of mercury. The values so obtained, given in Table 4, show that very little of the air damping is due to the kinetic energy acquired by the adjacent mass of air from the motion of the coil. Practically all of the damping due to air results from the viscosity, which is constant over a great range of pressures.

The values of Table 4 are plotted in Fig. 10. Upward from 600 lines the variation in the logarithmic decrement is practically proportional to the change in field strength, which indicates that for the higher intensities of field the greatest part of the damping is due to eddy currents in the wire of the coil.

Table 4

Intensity of Field	Log. Dec. Atmospheric Pressure	Log. Dec. 0.5 mm Pressure
0	.01089	.01045
151	.01234	.01153
403	.02305	.02291
605	.03458	.03394
903	.05763	.05680
1208	.08008	.07978
1500	.1088	.1082

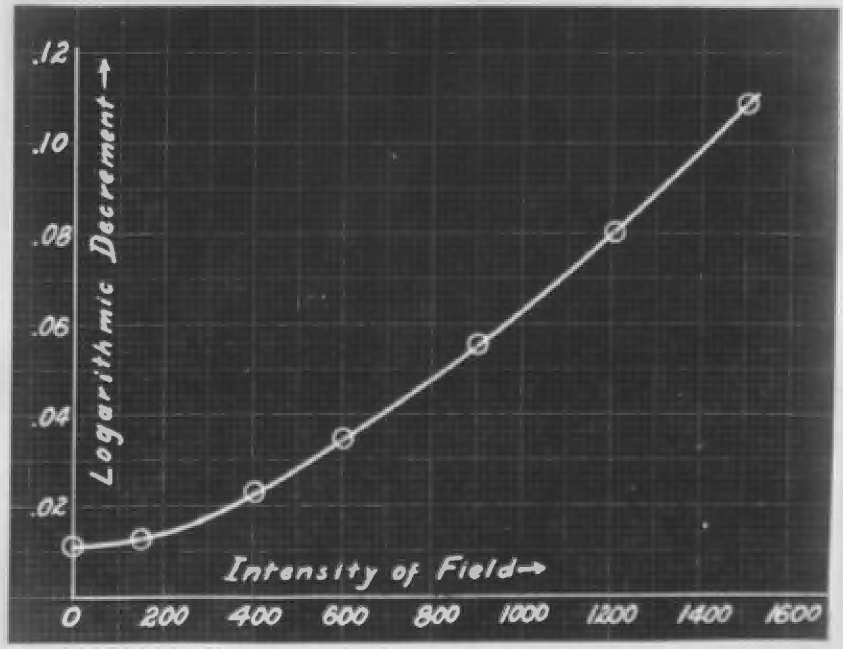
.12
.10
.08
.06
.04
.02
Logarithmic Decrement →
Intensity of Field →
0
200
400
600
800
1000
1200
1400
1600

Fig. 10.

SUMMARY

In the foregoing paper the following things are presented:

1. A general review of the theory of the motions which a galvanometer coil may have. An equation is given which shows that in a moderately damped coil the period is little affected by changes in the logarithmic decrement.

2. A method is given for measuring magnetic field intensities by means of a damped coil, the theory of which is based upon the general differential equation of the motion of a galvanometer coil. Results obtained by the damped coil method are compared with results by the ballistic method, and the conclusion is drawn from the comparison that the results obtained by the method here given may be relied upon for an accuracy of 1%. Advantages over the ballistic method are pointed out.

3. An equation is given for finding the size of wire to be used in an auxiliary damping rectangle

such that the coil to which the rectangle is attached shall be critically damped, or nearly so.

4. An equation is given for finding the resistance of a galvanometer coil by observing its logarithmic decrements when known resistances are connected in series with it.

5. A curve is given which shows that under certain conditions the period of a galvanometer coil may increase slightly with increasing weak magnetic fields. An explanation is offered, and an experiment described to show the explanation valid.

6. An experiment is described which shows that practically all the damping when a coil is swinging in air and zero field is due to viscosity of the air.

In conclusion I wish to acknowledge my indebtedness to Professor A. Zeleny for valuable suggestions he has given during the course of this work.

BIBLIOGRAPHY

Stansfield, Phil. Mag. 46, (5), 1898, p. 67. Reference to zero shift as being due to magnetic impurities in the galvanometer coil.

Ayrton and Mather, Phil. Mag. 46, (5), 1898, p. 351: "Galvanometers."

Diesselhorst, Ann. d. Phys. 9, 1902, p. 458: "Ueber ballistische Galvanometer mit beweglicher Spule." --Zschr. f. Instrumentenkunde 31, 1911, p. 247: "Ueber die Berechnung von Drehspulgalvanometern."

Jaeger, Zschr. f. Instrumentenkunde 23, 1903, pp. 261 and 353: "Das Drehspulgalvanometer im aperiodischen Grenzfall." -- Ann. d. Phys. 21, 1906, p. 64: "Ueber das Drehspulgalvanometer."

Stewart, O. M., Phys. Rev. 16, 1903, p. 158: "The Damped Ballistic Galvanometer."

White, Phys. Rev. 19, 1904, p. 305: "Sensitive Moving Coil Galvanometers." -- Phys. Rev. 23, 1906, p. 382: "Everyday Problems of the Moving Coil Galvanometer."

Zeleny, A., Phys. Rev. 23, 1906, p. 399: "On Precision Measurements with the Moving Coil Ballistic Galvanometer." -- Phys. Rev. 32, 1911, p. 297: "The Causes of Zero Displacement and Deflection Hysteresis in Moving Coil Galvanometers."

Zeleny and Hovda, Phys. Rev. 28, 1909, p. 277: "The Temperature Coefficients of the Moving Coil Galvanometer."